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Sampled-Data Models for Linear and Nonlinear Systems

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To Paz and Rosslyn

Preface

Most real-world systems evolve in continuous time. However, digital implementation is almost universally used in practice. Hence, a crucial ingredient in practical estimation and control is an understanding of the impact of sampling on continuous-time models and systems. In this context, the aim of this book is to reduce the gap between continuous-time and sampled-data systems theory. The subject of sampling is huge—no one book can cover all aspects. Thus, the book emphasises exact and approximate models for sampled-data systems. Questions such as the following will be addressed:

- *What can one say when the sampling rate is high relative to the dynamics of interest?*
- *Do natural convergence results apply as the sampling rate increases?*
- *Do there remain any special features of sampled systems which are not associated with underlying continuous systems?*

The authors' motivation for writing the book was threefold:

(i) Whilst most systems evolve in continuous time, all modern control and signal processing equipment is computer based. Hence, sampling arises as an inescapable aspect of all modern control and signal processing applications.
(ii) Sampling is, at first glance, a straightforward issue. However, on closer examination, if sampling is not treated properly, misleading or erroneous results can occur.
(iii) The authors have found that many aspects of sampling are not completely understood by engineers and scientists, even though these issues are central to many of the applications with which they deal.

The goal of this book is to provide a guide for students, practising engineers, and scientists who deal with sampled-data models. The book is intended to act as a catalyst for further applications in the area of nonlinear estimation and control.

Four classes of systems are treated:

(i) linear deterministic systems,
(ii) nonlinear deterministic systems,

(iii) linear stochastic systems, and
(iv) nonlinear stochastic systems.

Several applications are also presented. These applications embellish the core ideas by showing how they impact several important problems in signals and systems.

The book was written in Valparaíso (Chile) and Newcastle (Australia) during enjoyable collaborative visits by the authors. The book assembles contemporary work of the authors and others on sampled-data models. The authors hope that, by setting these ideas down in one place, we can instill confidence in readers dealing with sampled-data issues in real-world applications of signal processing and control.

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Valparaíso, Chile Juan I. Yuz
Newcastle, Australia Graham C. Goodwin
September 19, 2013

Contents

Part II Stochastic Systems

Symbols and Acronyms

$\langle \circ, \circ \rangle$	Inner product.
$\sim$	*Distributed as* (for random variables).
*	Complex conjugation.
^{T}	Matrix (or vector) transpose.
γ	Complex variable associated to the δ-operator.
Δ	Sampling period.
δ	Delta operator (forward divided difference).
$\delta(t)$	Dirac delta or continuous-time impulse function.
$\delta_K[k]$	Kronecker delta or discrete-time impulse function.
$\mu(t)$	Unitary step function or Heaviside function.
$\mu[k]$	Discrete-time unitary step function.
$\rho = \frac{d}{dt}$	Time-derivative operator.
ω	Angular frequency, in [rad/s].
ω_N	Nyquist frequency, $\omega_N = \frac{\omega_s}{2}$.
ω_s	Sampling frequency, $\omega_s = \frac{2\pi}{\Delta}$.
A, B, C, D	State-space matrices in continuous time.
$A_\delta, B_\delta, C_\delta, D_\delta$	State-space matrices in discrete time using the δ-operator (i.e., incremental models).
A_q, B_q, C_q, D_q	State-space matrices in discrete time using the shift operator q.
adj	Adjoint of a matrix.
ASZ	Asymptotic sampling zeros.
$\mathbb{C}$	Set of complex numbers.
$\mathcal{C}^n$	Space of functions whose first n derivatives are continuous.
CAR	Continuous-time auto regressive.
CSZ	Corrected sampling zeros.
CT	Continuous time.
CTWN	Continuous-time white noise.
det	Determinant of a matrix.
DFT	Discrete Fourier transform.
DT	Discrete time.
DTFT	Discrete-time Fourier transform.

DTWN	Discrete-time white noise.
$E\{\cdot\}$	Expected value.
ESD	Exact sampled-data (model).
$f(t)$	Continuous-time signal ($t \in \mathbb{R}$).
f_k or $f[k]$	Discrete-time signal or sequence ($k \in \mathbb{N}$).
$\mathcal{F}\{\cdot\}$	(Continuous-time) Fourier transform.
$\mathcal{F}^{-1}\{\cdot\}$	(Continuous-time) inverse Fourier transform.
$\mathcal{F}_d\{\cdot\}$	Discrete-time Fourier transform.
$\mathcal{F}_d^{-1}\{\cdot\}$	Discrete-time inverse Fourier transform.
FDML	Frequency-domain maximum likelihood.
FOH	First order hold.
$G(\rho)$	Deterministic part of a continuous-time system.
$G(s)$	Continuous-time transfer function (s-domain of the Laplace transform).
$G_\delta(\gamma)$	Discrete-time transfer function (γ-domain of the δ operator).
$G_q(z)$	Discrete-time transfer function (z-domain of the shift operator q).
GHF	Generalized hold function.
GSF	Generalized sampling filter.
$H(\rho)$	Stochastic part of a continuous-time system.
$h_g(t)$	Impulse response of a generalized hold or sampling function.
$\Im$	Imaginary part of a complex number.
I_n	Identity matrix of dimension n.
IV	Instrumental variables.
$\mathcal{L}\{\cdot\}$	Laplace transform.
$\mathcal{L}^{-1}\{\cdot\}$	Inverse Laplace transform.
ℓ_2	Space of square summable sequences.
$\mathcal{L}_2$	Space of square integrable functions.
LQ	Linear-quadratic (optimal control problem).
LS	Least squares.
MIFZ(D)	Model incorporating fixed zero (dynamics).
MIMO	Multiple-input multiple-output (system).
MIPZ(D)	Model incorporating parameterised zero (dynamics).
ML	Maximum likelihood.
MPC	Model predictive control.
$\mathbb{N}$	Set of natural numbers (positive integers).
$N(\mu, \sigma^2)$	Normal (or Gaussian) distribution, with mean μ and variance σ^2.
NMP	Non-minimum phase.
$\mathcal{O}(\Delta^n)$	Function *of order* Δ^n.
PEM	Prediction error method(s).
PSD	Power spectral density.
QP	Quadratic programming.
q	Forward shift operator.
$\Re$	Real part of a complex number.

$\mathbb{R}$	Set of real numbers.
s	Complex variable corresponding to the Laplace transform.
SD	Sampled data.
SDE	Stochastic differential equation.
SDR (or **SDRM**)	Simple derivative replacement (model).
SISO	Single-input single-output (system).
TTS	Truncated Taylor series.
$\mathbb{Z}$	Set of integer numbers.
$\mathcal{Z}\{\cdot\}$	$\mathcal{Z}$-transform.
$\mathcal{Z}^{-1}\{\cdot\}$	Inverse $\mathcal{Z}$-transform.
z	Complex variable corresponding to the $\mathcal{Z}$-transform of the shift operator q.

Chapter 1
Introduction

Models for continuous-time dynamical systems typically arise from the application of physical laws such as conservation of mass, momentum, and energy. These models evolve in continuous time and take the form of linear or nonlinear *differential* equations. In practice, however, these kinds of models do not tell the complete story when the system is connected to digital devices. For example, when digital controllers have to act on a real system, this action can be applied (or updated) only at specific time instants. Similarly, if data is collected from a given system, the data is typically only recorded (and stored) at specific sampling instants. As a consequence, *sampling* arises as a cornerstone problem in all aspects of modern estimation and control theory.

In this context, the current book develops *sampled-data models* for linear and nonlinear systems. The focus is on describing, in discrete time, the relationship between the input signal and the samples of the continuous-time system output. In particular, issues such as the *accuracy* of sampled-data models, the *artefacts* produced by particular sampling schemes, and the relationship between the sampled-data model and the underlying continuous-time system are studied.

The sampling process for a continuous-time system is represented schematically in Fig. 1.1. There are four basic elements shown in the figure. Each of these elements plays a role in determining the appropriate discrete-time input-output description:

- *The hold*: The hold is used to convert a discrete-time sequence $\{u_k\}$ into a continuous-time input. A very commonly used hold in practical systems is the zero-order hold, where

$$u(t) = u_k \quad \text{for } k\Delta \leq t < (k+1)\Delta \tag{1.1}$$

  where Δ is the sampling time.
- *The physical system* The system will typically be described by a set of linear or nonlinear differential equations.
- *The anti-aliasing filter (AAF)*: The anti-aliasing filter prepares the continuous-time signal prior to taking samples.

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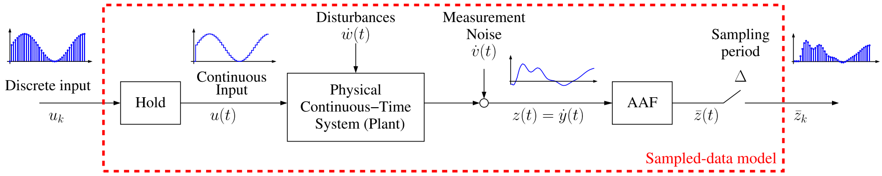

Fig. 1.1 Scheme of the sampling process of a continuous-time system

- *The sampler*: The sampler creates a discrete-time sequence $\{z_k\}$ by instantaneous sampling, i.e.,

$$\bar{z}_k = \bar{z}(k\Delta) \tag{1.2}$$

For linear systems, it is possible to obtain exact sampled-data models from the sampling scheme shown in Fig. 1.1. In particular, given a deterministic continuous-time system, it is possible to obtain a discrete-time model which replicates the sequence of output samples. In the stochastic case, where the input of the system is assumed to be a *continuous-time white noise* process, a sampled-data model can be obtained such that its output sequence has the same second order properties as the continuous-time output at the sampling instants. Sampled models for nonlinear systems are harder to compute. For nonlinear deterministic systems, sampled-data models can be obtained such that the system output is reproduced up to some level of accuracy. In the nonlinear stochastic case, approximate sampled-data models can be obtained which approximate the stochastic properties of the output samples. Both linear and nonlinear *deterministic* systems are treated in Part I of the book, and linear and nonlinear *stochastic* systems are treated in Part II. Embellishments and extensions are presented in Part III.

Sampled-data models for continuous-time systems are used in many contexts, e.g., control, simulation, and estimation of system parameters (system identification). Most of the existing literature regarding discrete-time (and, thus, sampled-data) systems has traditionally expressed these models in terms of the shift operator q and the associated $\mathcal{Z}$-transform. However, when using this kind of model, it is not easy to relate the results to the continuous-time case. This is especially true when the sampling period is small. The inter-relationship between sampled-data models and their underlying continuous-time counterparts is more easily understood in the unified framework facilitated by the use of *incremental* models. In particular, sampled-data models *rewritten* in incremental form explicitly include the sampling period Δ in such a way that, when the sampling period is decreased, the underlying continuous-time system representation is recovered. In the same fashion, most discrete-time and continuous-time results in control, estimation, and system identification can be understood in a common framework when sampled-data models are expressed in incremental form.

It is also important to recall that, when using discrete-time models to represent continuous-time systems, there is *loss of information*. In the time domain, the intersample behaviour of signals is unknown, whereas in the frequency domain, high frequency signal components will fold back to low frequencies, making them impossible to distinguish. The loss of information is central to the difference between sampled-data and continuous-time systems. In Chap. 2, sampling and reconstruction of band-limited signals will be discussed. In Chap. 3, sampled-data models for linear deterministic systems having zero order hold inputs will be developed.

For any non-zero sampling period, there will always be a *difference* between the sampled-data model and the underlying continuous-time description. For example, discrete-time models have, in general, more zeros than the original continuous-time system. These extra zeros, called *sampling zeros*, are a result of the frequency folding effect due to the sampling process. This aspect of sampling will be discussed for deterministic systems in Chap. 5, and for linear stochastic systems in Chap. 14. The presence of sampling zeros (and their asymptotic behaviour) is determined by the continuous-time system relative degree. However, relative degree may be affected by high frequency modelling errors, even beyond the sampling frequency. Thus, robustness issues are discussed in Chaps. 7 and 15.

The book repeatedly highlights the issues and assumptions related to the use of sampled-data models. Some examples of these issues are as follows:

- Sampled-data characteristics depend not only on the continuous-time system but also on the sampling process itself. Indeed, for linear systems, the discrete-time poles depend on the continuous-time poles and the sampling period, whereas the zeros depend on the choice of the hold and the sampling devices.
- The effects of sampling *artefacts*, such as sampling zeros, play an important role in describing accurate sampled-data models. This applies both to exact sampled models for linear systems and to approximate sampled models for nonlinear systems.
- Any sampled-data description is based on some kind of *model* of the true continuous-time system. Modelling errors will usually be important at high frequencies due to the presence of unmodelled poles, zeros, or unmodelled time delays in the continuous-time system. This means that continuous-time models usually must be considered within a *bandwidth of validity* for the system.
- For stochastic models, the unknown input is assumed to be a (filtered) continuous-time white noise process. This is a mathematical abstraction that usually does not correspond to physical reality. However, it can be approximated to any desired degree of accuracy by conventional stochastic processes with broad-band spectra. This means that stochastic systems must be treated carefully. For example, the non-ideal nature of the noise can be thought of as a form of high frequency modelling error. How this aspect affects the use of sampled-data models will be discussed in Chap. 19.

The focus throughout the book is to provide answers to questions of the following type:

- When using sampled-data models, what characteristics are inherent to the underlying continuous-time system and what characteristics are a consequence of the sampling process itself?

- Is it reasonable to assume that, as the sampling period is decreased, the sampled-data model becomes indistinguishable from the underlying continuous-time system? How does this convergence occur?
- What issues are important when using a discrete-time model to represent a system?
- How do the results on sampling for linear systems apply to nonlinear systems?

Answers will be provided to these questions as the book evolves. As a preliminary step, in the next chapter, background material on sampling of signals and Fourier analysis is provided.

Part I
Deterministic Systems