BIFURCATIONS & INSTABILITIES IN GEOMECHANICS

PROCEEDINGS OF THE INTERNATIONAL WORKSHOP ON BIFURCATIONS & INSTABILITIES IN GEOMECHANICS, IWBI 2002, 2-5 JUNE 2002, MINNEAPOLIS MN, USA

Bifurcations & Instabilities in Geomechanics

Edited by

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Cover: Schematic diagram of the University of Minnesota Plane Strain Apparatus; results from a plane-strain test on Berea sandstone (from the Geomechanics Laboratory, Department of Civil Engineering, University of Minnesota).

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Preface

Several seminal papers on localization and related instabilities were published in the 1970's and early 1980's, and bifurcation theory was becoming a topical subject among a small group of engineers in the field of geomechanics. To assemble this modest community and provide a forum for discussion, the first International Workshop on Localization of Soils was held in Karlsruhe, Germany, February 1988. It was followed by the second international workshop in Gdansk, Poland, September 1989. The first two events emphasized fundamental aspects of bifurcation theory applied to soil mechanics. The third international workshop was held in Aussois, France, September 1993, and the topics were extended to include rock mechanics. The sites for the fourth and fifth workshops were Gifu, Japan (September 1997) and Perth, Australia (November 1999). The University of Minnesota was selected as the host for the sixth workshop, which was held on June 2-5, 2002 and named the International Workshop on Bifurcations & Instabilities in Geomechanics (IWBI 2002).

The common themes throughout the past workshops have been modeling of bifurcation, failure of geomaterials and structures, advanced analytical, numerical and experimental techniques, and application and development of generalized continuum models. The objective of IWBI 2002 was to continue these themes by bringing together international researchers and practitioners dealing with bifurcations and instabilities in geomechanics, specifically to collect and debate the developments and applications that have taken place the past few years. The scope of the workshop was broadened to include not only localization and other bifurcation modes, but also related instabilities, for example, due to softening (inherent or geometric) and loading conditions (static rather than kinematic), among others. This was reflected in the name "International Workshop on Bifurcations and Instabilities in Geomechanics." The venue for IWBI 2002 was the campus of St. John's University in Collegeville, Minnesota, about 100 km from the University of Minnesota. Surrounded by woodlands and lakes, St. John's provided a wonderful environment for discussions and interactions.

IWBI 2002 was attended by 74 participants representing 15 countries; 45 presentations were given over three days. The papers in this book are a sampling of the presentations. The development of the program was aided by the Scientific Committee consisting of F. Darve, H. Muhlhaus, F. Oka, I. Vardoulakis, and D. Muir Wood, with input from the Advisory Committee consisting of T. Adachi, J.P. Bardet, R. de Borst, J. Desrues, G. Gudehus, D. Kolymbas, R. Michalowski, Z. Mroz, R. Nova, S. Pietruszczak, and J. Rudnicki. Their efforts contributed immensely to the workshop.

IWBI 2002 was sponsored by the National Science Foundation, Civil & Mechanical Systems Division, Geomechanics and Geotechnical Systems. Additional support was provided by (in alphabetical order) BP America, Houston, TX; Itasca Consulting Group, Minneapolis, MN; MTS Systems, Eden Prairie, MN; Schlumberger Cambridge Research, Cambridge, UK; Schlumberger Engineering Applications, Sugar Land, TX; Shell International Exploration and Productions, Houston, TX; University of Minnesota, Institute of Technology and Department of Civil Engineering, Minneapolis, MN.

Joseph F. Labuz & Andrew Drescher

Organisation

The June 2-5, 2002 International Workshop on Bifurcations & Instabilities in Geomechanics was sponsored by the National Science Foundation, Civil & Mechanical Systems Division, Geomechanics and Geotechnical Systems. Additional support was provided by (in alphabetical order) BP America, Houston, TX; Itasca Consulting Group, Minneapolis, MN; MTS Systems, Eden Prairie, MN; Schlumberger Cambridge Research, Cambridge, UK; Schlumberger Engineering Applications, Sugar Land, TX; Shell International Exploration and Productions, Houston, TX; University of Minnesota, Institute of Technology and Department of Civil Engineering, Minneapolis, MN.

Scientific Committee: F. Darve (France), H. Muhlhaus (Australia), F. Oka (Japan), I. Vardoulakis (Greece), D. Muir Wood (UK)

Advisory Committee: T. Adachi, (Japan), J.P. Bardet (USA), R. de Borst (The Netherlands), J. Desrues (France), G. Gudehus (Germany), D. Kolymbas (Austria), R. Michalowski (USA), Z. Mroz (Poland), R. Nova (Italy), S. Pietruszczak (Canada), J. Rudnicki (USA)

1. Critical states and bifurcation conditions

The failure concept in Soil Mechanics revisited

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ABSTRACT: When applied to porous materials characterised by a non- associate flow rule such as soils, the apparently self evident concept of failure becomes less neat. Although it is possible to define a convenient limit locus beyond which stress states are not feasible for a given material, unlimited strains may occur even for stress states within such locus, provided convenient loading conditions are imposed and controlling parameters appropriately chosen.

It is shown that such type of failures can occur in the hardening regime, provided the flow rule is non-associate and the stiffness matrix is not positive-definite. Failures due to loss of load control in undrained tests or shear or compaction band occurrence are examples.

The related concepts of hardening ad softening are discussed. It is shown that, contrary to primitive expectations, hardening may occur even with decreasing deviator stress, while softening may be associated to a load increment.

1 INTRODUCTION

In elementary Soil Mechanics books, the shear strength of geomaterials is defined in terms of the Coulomb-Mohr failure condition. In the principal stress space, the failure locus is the lateral surface of a pyramid of equation

$$\left|\sigma_i' - \sigma_j'\right| = 2c' \cos\phi' + (\sigma_i' + \sigma_j') \sin\phi' \tag{1}$$

where c' and ϕ' are the cohesion and the friction angle of the soil and σ_k' are the effective principal stresses.

This implies that the stress states for which the corresponding stress points lie on that surface are 'at failure', that means that unlimited strains can take place under constant stress. As a consequence, no stress point outside the locus can be reached. On the contrary, all stress states strictly within this locus are 'safe', which means that sufficiently small stress perturbations generate small strains only.

Unfortunately such an elementary concept of failure is contradicted by experimental evidence: 'failure', i.e. unlimited strains, can occur, under special conditions, even when the stress point is within the domain usually considered as 'safe'.

In order to show that, it is enough to consider a specimen of loose saturated sand in axisymmetric (so called 'triaxial') conditions. If a drained test at constant cell pressure is performed, the deviatoric stress, q, increases with axial strain, ε_a, approaching a horizontal asymptote, as shown in Fig. 1. The stress ratio

η_f, defined as the ratio between q and the isotropic effective pressure, p', at this asymptotic level, is independent of the lateral confining pressure and corresponds to a friction angle of about 30°.

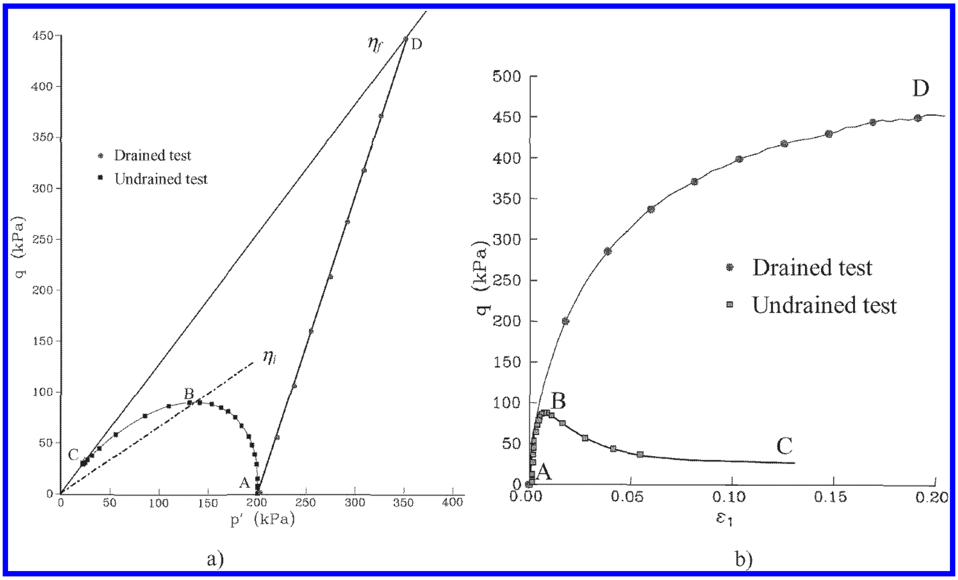

Figure 1. a) stress paths of drained and undrained triaxial tests on loose sand specimens b) corresponding stress-strain relationship

If, on the contrary, we perform an undrained test under displacement control and constant confining pressure, the pore water pressure increases with increasing strains, while the stress deviator reaches a peak and then dramatically decreases, eventually approaching a horizontal asymptote from above. The stress ratio corresponding to this final stage is again η_f, while the stress ratio at which the peak occurs, η_i, is much lower and corresponds to a mobilized friction angle of about 16°. Since strains at peak are still very small, the area of the right section of the specimen is almost the same as the initial one. Deviator stresses vary proportionally to the vertical load, therefore, and a peak in q corresponds to a peak in the axial load. It is clear then that, if an undrained test is performed on a theoretically identical specimen by monotonically increasing the vertical load, all the rest being unchanged, when the stress ratio reaches η_i, a loss of control of the specimen occurs and theoretically unlimited strains take place.

In geotechnical engineering, the peak in the undrained test is often considered to be a point of hardening-softening transition. This fact has a serious shortcoming, however. Softening is certainly associated to a loss of load control, as that shown by the experimental results, but also to a negative hardening modulus, if the terms hardening and softening are used as in the theory of plasticity. But this is not the case at the undrained peak. For an elastoplastic strainhardening material, the hardening modulus H is defined as follows:

$$d\varepsilon_{ij}^{p} = \frac{1}{H}\frac{\partial g}{\partial \sigma'_{ij}}\frac{\partial f}{\partial \sigma'_{hk}} d\sigma'_{hk} \tag{2}$$

$$H \equiv -\frac{\partial f}{\partial \psi_l}\frac{\partial \psi_l}{\partial \varepsilon_{rs}^{p}}\frac{\partial \varepsilon_{rs}^{p}}{\partial \sigma'_{rs}} \tag{3}$$

where g is the plastic potential, f is the loading function and ψ_l are the internal variables.

As it can be derived from Equation 3, the hardening modulus depends only on the state of stress and the previous plastic strain history of the material, but does not depend on the stress rate direction. The hardening modulus at the peak point is therefore the same in drained and undrained conditions. Since it is positive in the former case, it must be so also in the latter one.

Di Prisco et al. (1995) experimentally demonstrated this fact. Fig 2 shows the behavior of a loose sand specimen that is first loaded in undrained conditions up to the peak. Then, after pore water pressure equalization, drainage is allowed for and the axial load increased. As it is apparent, the behavior is typically hardening and no phenomenon of loss of control occurs. The response of the material depends therefore on the stress rate direction and can be associated either to an increase or a decrease of the stress deviator, depending on the kinematic control (volumetric strain allowed for or not). The hardening modulus must be positive in both cases, however, since it is independent of the stress rate direction and load control is possible in the drained case.

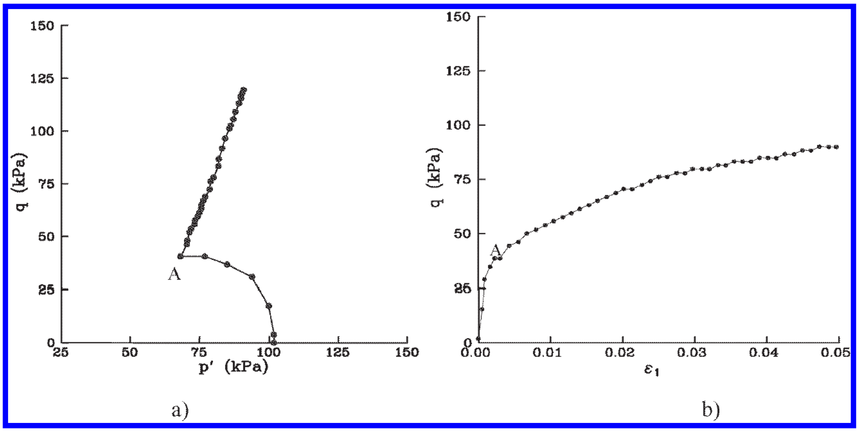

Figure 2. a) stress path and b) stress-strain relationship for a specimen loaded in undrained conditions up to peak and then allowed to drain for further axial loading – After di Prisco et al. (1995)

We can therefore conclude that, under special loading conditions and type of load control, failure of the specimen can occur at a stress point that is well within what is normally considered as a 'safe' domain. This is not associated to a hardening-softening transition, but it must be related to other properties that are hidden in the constitutive relationship of the material.

In the following, the conditions for such failures to occur will be recalled. It will be shown that failure can be associated to the concept of loss of control. Theoretically unlimited strain rates can take place, in fact, as a consequence of infinitesimal loading perturbations. Failure within the 'safe' domain is therefore associated to a homogeneous bifurcation, such as the unlimited pore water pressure generation in undrained tests. Even the occurrence of shear or compaction bands can be treated in this framework. This will eventually lead us to reconsidering the meaning of hardening and softening concepts, as traditionally perceived. It will be shown that, contrary to primitive expectations, hardening may occur even with decreasing deviator stress, while softening may be associated to a load increment.

2 CONDITIONS FOR LOSS OF CONTROL ('LATENT INSTABILITY')

The analysis that will be presented here (Nova (1989), (1994)) is based on the hypothesis that a soil specimen of finite size under external loading is in a uniform state of stress and strain. This assumption is usually made for the interpretation of actual test results. Furthermore it will be assumed that strains are small, so that stresses are proportional to applied loads and strains to displacements. The analysis of the load-displacement behavior for a specimen of finite size is therefore equivalent to that of the stress-strain relationship at a constitutive level.

In matrix terms, the latter can be expressed as:

$$\dot{\sigma}' = D\dot{\varepsilon} \tag{4}$$

Under stress (load) control, unlimited strain (displacement) rates occur when

$$\det D = 0 \tag{5}$$

Equation 5 gives what is usually considered as the failure condition and, for a strain hardening material, the state for which this occurs is characterized by the nullity of the hardening modulus. The locus in the stress space for which Equation 5 is fulfilled can be also considered as a limit locus, since a virgin soil cannot bear a stress state beyond it.

In geotechnical tests, however, usually the controlling parameters are some stress and some strain components. The stress-strain relationship can be therefore rearranged, in order to have on the l.h.s. all the controlling parameters, for instance:

$$\begin{Bmatrix} \dot{\sigma}'_1 \\ \dot{\varepsilon}_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \dot{\varepsilon}_1 \\ \dot{\sigma}'_2 \end{Bmatrix} \tag{6}$$

where $\dot{\sigma}'_i$ and $\dot{\varepsilon}_j$ are vectors of n and $6 - n$ components, respectively, (with $0 \leq n \leq 6$), while A_{ij} are submatrices that are obtained by rearranging the stiffness matrix D. In this case, the condition for the occurrence of unlimited strains (and stresses) is clearly

$$\det A = 0 \tag{7}$$

It can be shown (Nova (1989), (1994)) that this occurs when

$$\det D_{11} = 0 \tag{8}$$

where D_{11} is the submatrix connecting $\dot{\sigma}'_1$ to $\dot{\varepsilon}_1$. In other words, whenever a minor of the stiffness matrix is nil, there exists a particular set of controlling parameters for which the complementary parameters of stress and strain can increase indefinitely.

Since, in many instances, strain and stress rates are not controlled directly, but the value of a linear combination of them is imposed (generalized stresses and strains, such as volumetric strain or deviator stress), Equation 8 can be generalized to

$$\det \Delta_{11} = 0 \tag{9}$$

where Δ is a matrix connecting generalized stress and strain rate vectors.

It can be shown further that if the stiffness matrix is positive definite, neither one of Equations 5, 8 and 9 can be fulfilled. Unlimited strain rates cannot take place under any loading program. On the other hand, if the stiffness matrix is positive semi-definite, there exists one particular loading program, in terms of generalized stress-strain, for which loss of control occurs. For stress states beyond that level, there is an entire cone of possible load paths for which such a possibility exists (Imposimato and Nova (2001)).

The stiffness matrix of a material obeying an associate flow rule is symmetric and becomes positive semi-definite when the limit locus is achieved. All types of load or displacement control path, even if generalized, give rise to limited responses for small perturbations below that locus. No failure of the type discussed so far is possible, therefore, in the hardening regime ($H > 0$). If the flow rule is non-associate, however, the stiffness matrix is not symmetric, the loss of positive definiteness occurs in the hardening regime and load programs for which unlimited strains occur within the usually considered safe domain are possible (Nova (1989), (1994)).

The occurrence of such failures in the hardening regime is indeed a clue of the fact that the flow rule is non-associate.

Whenever either one of Equations 5,8 or 9 is fulfilled, infinitely many eigen-solutions are possible. On the one hand this implies that the fulfillment of any of these Equations corresponds to a state for which lack of uniqueness of the incremental response takes place. On the other one, this also implies that solutions are possible only in the case load increments are given in a very special way. If not, no solution exists. For instance, when Equation 8 is fulfilled, it is possible an infinity of eigen-solutions of the type

$$\dot{\sigma}_2' = \beta D_{21}\dot{\varepsilon}_1^* + D_{22}\dot{\varepsilon}_2 \tag{10}$$

where β is an indefinite scalar and $\dot{\varepsilon}_1^*$ is the eigen-vector of matrix D_{11}, when the load increment is such that

$$\dot{\sigma}_1' = D_{12}\dot{\varepsilon}_2 \tag{11}$$

This condition is similar to that occurring when a 'latent instability' analysis of structures under load is made (Ziegler (1968)). In that case, infinitely many configurations of the structure are possible, under the same 'critical' load. By analogy, we shall denote the aforementioned conditions as 'latent instability' conditions.

3 EXAMPLES OF LOSS OF CONTROL IN THE HARDENING REGIME

The occurrence of a peak in the undrained test of Fig 2 can be easily predicted in this framework. In an axisymmetric test, the constitutive law can be condensed as follows:

$$\begin{Bmatrix} \dot{\varepsilon}_v \\ \dot{\varepsilon}_d \end{Bmatrix} = \begin{bmatrix} C_{pp} & C_{pq} \\ C_{qp} & C_{qq} \end{bmatrix} \begin{Bmatrix} \dot{p}' \\ \dot{q} \end{Bmatrix} \tag{12}$$

where ε_v and ε_d denote the volumetric and the deviatoric strain, respectively, defined as:

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \tag{13}$$

$$\varepsilon_d = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{14}$$

and C_{ij} are the elements of the compliance matrix.

In the case considered, the controlling variables are the volumetric strain, that must be constant, and the axial load. At stress deviator peak, the volumetric compliance C_{pp} must be zero, to comply with the first of Equations 12. At peak, the stress strain relationship is therefore such that:

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} -C_{pq}C_{qq}^{-1}C_{qp} & C_{pq}C_{qq}^{-1} \\ -C_{qq}^{-1}C_{qp} & C_{qq}^{-1} \end{bmatrix} \begin{Bmatrix} \dot{p}' \\ \dot{\varepsilon}_d \end{Bmatrix} \tag{15}$$

The determinant of the matrix of Equation 15 is zero and unlimited deviatoric strain and pore pressure rates, $\dot{u}$, can take place:

$$\dot{p}' = -\dot{u} = -\varphi \tag{16}$$

$$\dot{\varepsilon}_d = -C_{qp}\varphi \tag{17}$$

where φ can be any (positive) scalar.

Conversely Equation 18:

$$C_{pp} = C_{ijhk}\delta_{ij}\delta_{hk} = 0 \tag{18}$$

gives the condition for a peak of the deviator stress and consequently for latent instability (loss of control) of a load controlled test. It can be shown that Equation 18 is the equation of a straight line passing through the origin of axes in the p', q plane, known as Lade instability line (Lade (1992)).

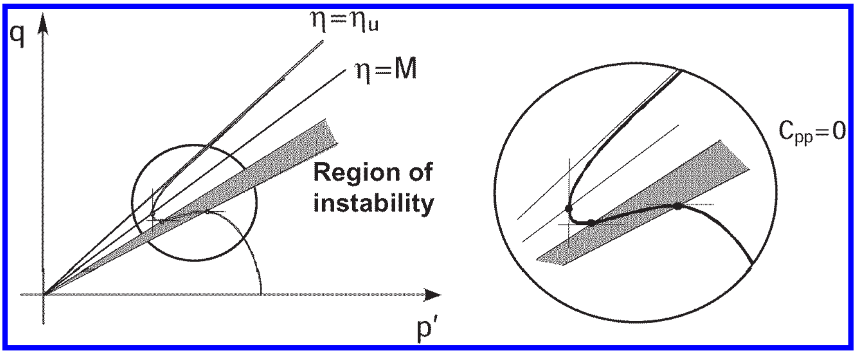

Figure 3. Region of instability in the 'triaxial' plane under load control in undrained conditions

It is perhaps of some interest to note in passing that the region of instability is of limited extension, as shown in Figure 3 for a medium dense sand. This region is in fact confined by two straight lines, passing through the origin. Both fulfill Equation 18.

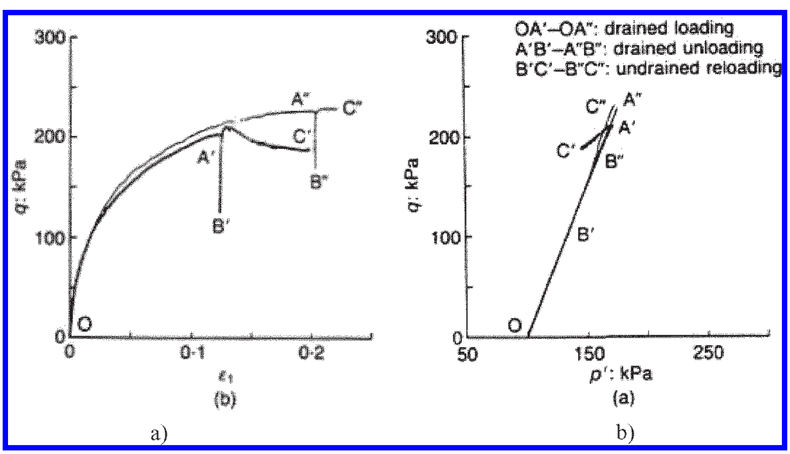

Figure 4. a) stress paths and b) stress-strain relationships for loose Hostun sand specimens preloaded in drained conditions, unloaded and reloaded in undrained conditions. After di Prisco et al. (1995)

If the stress path by-passes such a region, by drained preloading, for instance, no control loss occurs in load controlled undrained reloading. Figure 4 shows in fact the experimental results conducted on two theoretically identical specimens of loose Hostun sand (di Prisco et al. (1995)). They differ in that the drained preloading is larger for the second case (OA”) than in the first one (OA’), A’ being inside the instability region, while A” is above it. It is apparent from Figure 4 that the undrained reloading branches show a very different behaviour. A peak of the deviator stress is observed for path B’C’, implying instability under load control, while a monotonic increase of the deviator stress is recorded along path B”C”.

Another case of loss of control occurs when the controlling quantities are one principal strain rate, say $\dot{\varepsilon}_1$, and two principal stress rates, say $\dot{\sigma}'_2$ and $\dot{\sigma}'_3$. By grouping the last two terms and the corresponding strain rates in vectors $\dot{\sigma}'_\alpha$ and $\dot{\varepsilon}_\alpha$, respectively, after variable rearrangement, provided the stiffness D_{11} is not nil, the constitutive law can be rewritten as

$$\begin{Bmatrix} \dot{\varepsilon}_1 \\ \dot{\sigma}'_\alpha \end{Bmatrix} = \frac{1}{D_{11}} \begin{bmatrix} 1 & -D_{1\alpha} \\ D_{\alpha 1} & D_{\alpha\alpha}D_{11} - D_{\alpha 1}D_{1\alpha} \end{bmatrix} \begin{Bmatrix} \dot{\sigma}'_1 \\ \dot{\varepsilon}_\alpha \end{Bmatrix} \tag{19}$$

According to Schur (1917) theorem, when

$$\det D_{\alpha\alpha} = 0 \tag{20}$$

the determinant of the matrix of Equation 12 is zero. Therefore, in general, the incremental solution does not exist and we cannot assign arbitrarily the increments at the l.h.s. Infinite eigen-solutions exist however, for the following load program

$$\dot{\sigma}'_\alpha = D_{\alpha 1}\dot{\varepsilon}_1 \tag{21}$$

Infinite solutions are therefore associated to the same load increment and a homogeneous bifurcation takes place. Note further that it can be shown that, when Equation 8 is fulfilled, also the complementary coefficient of the compliance matrix C_{11} is zero.

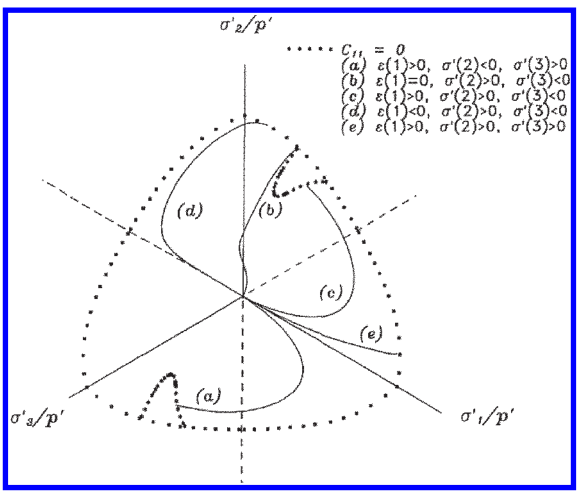

Figure 5. Stress paths in the deviatoric plane for different loading programs in which one principal strain rate and two principal stress rates are controlled. Homogeneous bifurcation loci are starred (after Imposimato and Nova (1998))

For instance, for a convenient set of parameters of the constitutive model ‘Sinfonietta Classica’ (Nova (1988)), the starred loci for which Equation 13 is satisfied are shown in Figure 5. It is also shown that out of the five stress paths considered, two touch the loci. A loss of control trol and homogeneous bifurcation occur therefore at such points, under those particular loading programs.

The framework of this analysis can be used also for determining the loci for which a drained shear band takes place (Nova (1989), Imposimato and Nova (1998)). It can be proven that, for an isotropic material, in the principal stress space, such loci coincide with those shown in Figure 5 (plus the symmetric loci obtained by permutation of indices). At the same stress point, therefore, both homogeneous and inhomogeneous (shear band) bifurcation can take place. The difference is in that homogeneous bifurcation is possible only under the very special load paths illustrated in Figure 5, whilst a shear band can occur for any type of load increment, because two different fields of strain and stress rates can coexist *en echelon* violating neither equilibrium nor compatibility. For this reason, in practice, in plane strain problems shear bands are more frequently observed than homogeneous bifurcation.

4 SOFTENING AND LOSS OF CONTROL IN OEDOMETRIC TESTS

Loss of control can occur, in special conditions, even in the most traditional of the geotechnical tests: the confined compression, so called oedometric, test. It will be shown in fact, by means of an elastoplastic strainhardening constitutive model, that during this test, an overconsolidated or a cemented soil specimen can reach the limit condition, soften and then harden again, provided the initial overconsolidation or degree of cementation are large enough. Furthermore, it will be shown that, for a particular choice of the constitutive parameters, appropriate for strong but brittle intergranular bonding and large initial porosity, vertical collapse of the specimen can take place, with possible formation of compaction bands and loss of load control.

Consider first the model for a cemented soil or a soft rock (Nova (1992)) where the bond strength is characterized by two fixed parameters p_t and p_m .The size of the elastic domain is controlled by p_s, that is a hardening variable, depending on the plastic strains experienced by the material. A picture of the initial elastic domain is shown in Figure 6. Assume, for the sake of simplicity, that the flow rule is associated and that hardening depends only on volumetric strains, as in Cam Clay (Schofield and Wroth (1968)). Compaction therefore corresponds to an increase in p_s and conversely dilation to its decrease.

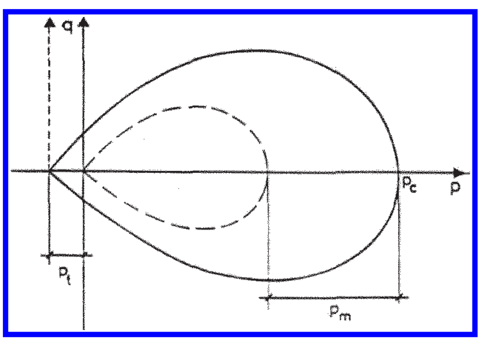

Figure 6. Initial elastic domain for a cemented material – after Nova (1992)

A typical stress path experimentally obtained in an oedometric test on cemented geomaterials, such as chalk (Leddra (1988)) or calcarenite (Coop and Atkinson (1993)), is shown in Figure 7a. The model predicts a similar trend. At the beginning the behavior is elastic, since the origin belongs to the initial elastic domain. The stress path is therefore given by a straight line, whose slope depends on the Poisson's ratio, only. The yield locus is reached at a point for which the horizontal component of the vector normal to it is directed in the negative sense of the hydrostatic axis, see Figure 7b. Plastic dilation is predicted, therefore, and consequently, for the assumed hardening law, softening of the material occurs.

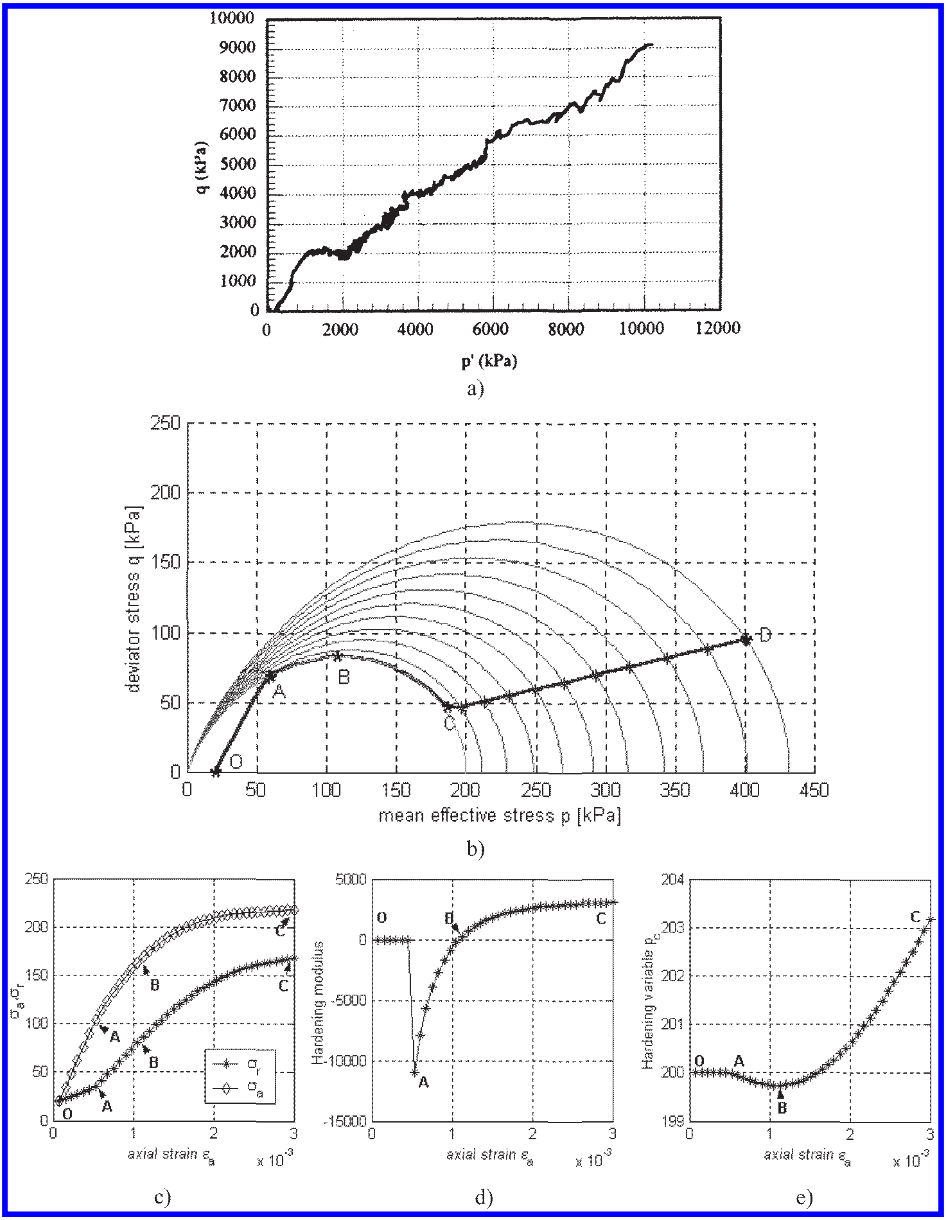

Figure 7. Oedometric test on a cemented geomaterial a) experimental (calcarenite) stress path- after Coop and Atkinson b) Calculated stress path c) stress-strain law d) value of the hardening modulus e)value of the hardening variable – After Castellanza (2002)

From the standpoint of the theory of plasticity, the specimen 'fails' at this point. The stress path is directed within the initial elastic domain and the hardening variable decreases together with its size, Figure 7e. The deviator stress increases, however, as well as the external loading. Full controllability of the test is possible, therefore, even under load control.

It is quite interesting to note that, despite the deviator stress increases, the hardening modulus is negative, so that softening occurs, Fig 7d. On the other hand, softening does not imply loss of controllability of the test, for the special loading program we are following, since vertical stress increases monotonically, Figure 7c. On the contrary, if the load program would be stopped at any moment between points A and B of Figure 7 and a different load increment, say an increase in vertical load at constant horizontal stress, would be imposed, the specimen would instantaneously collapse.

As point B is reached, the normal to the yield locus is directed towards the positive sign of the isotropic axis. Plastic compression takes place as well as hardening, for the assumed hardening and plastic flow rules. Not withstanding that, the deviator stress decreases. The stress path is obliged to follow more or less the shape of the yield locus, since the elastic strains are small and the zero lateral strain condition imposes that the plastic ones are of the same order of magnitude. Only when it is achieved a stress condition for which the normality rule determines plastic strain rates that are compatible with the oedometric condition, strains can occur freely, the stress path can leave the yield locus shape and the deviator stress can increase again.

Remarkably, similar results can be obtained by using a basic model as the original Cam Clay for a preconsolidated material with an overconsolidation ratio high enough to render the specimen 'dryer than critical', i.e. heavily overconsolidated.

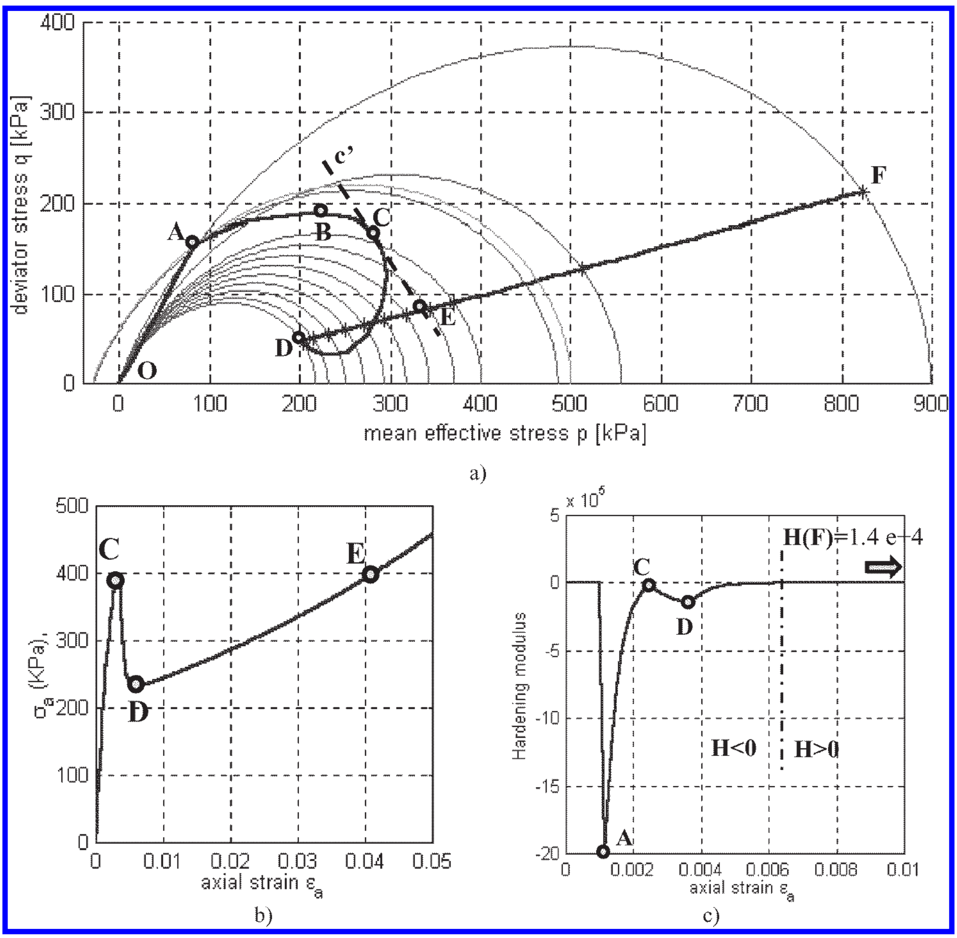

Figure 8. Oedometric test on a cemented geomaterial with bond degradation a) Calculated stress path b) axial stress strain relationship c)value of the hardening modulus –After Castellanza (2002)

Consider now a material whose bonds are brittle. Yielding is therefore associated to a decrease of the value of the variables p_t and p_m. In this case it is possible to have loss of load control even in the oedometric test. Figure 8 shows in fact the calculated results for a material characterized by convenient constitutive parameters. In this case the hardening parameter is given by the sum of three contributions (Gens and Nova (1993)):

$$H = H_s + H_t + H_m \tag{22}$$

each of which is related to the corresponding hardening variable p_s, p_t, p_m. While the first contribution can be either positive or negative, the other two are always negative. Softening can occur even in the region where the material is compacting, therefore, Figure 8c. The axial stress strain law shows a marked peak, implying that load control is impossible, beyond the peak. The calculated results were in fact obtained by imposing full kinematic control.

In such a test, in a specimen of finite size, but uniform state of stress and strain, at peak a bifurcation can take place without violating equilibrium nor compatibility. The specimen can be seen in fact as composed of layers, which are subject to the same vertical stress decrease but different horizontal stress variation, one layer undergoing elastic unloading while the adjacent one suffers elastoplastic loading, as shown in Figure 9. Such layers have been actually observed in soft rock specimens under compression (Olsson (1999)). Rudnicki (2002), following the same path of reasoning of Rudnicki and Rice (1975) for shear bands, has given the mathematical condition for the occurrence of compaction bands, i.e

$$D_{ijhk} n_i n_j n_h n_k = 0 \tag{23}$$

where D_{ijhk} is the stiffness tensor and n_i are the director cosines with respect to the reference axes of the normal n to the plane of the band.
According to tensor transformation rules, the l.h.s. of Equation 16 gives the stiffness coefficient D_{nnnn}. Equation 16 becomes therefore:

$$D_{nnnn} = 0 \tag{24}$$

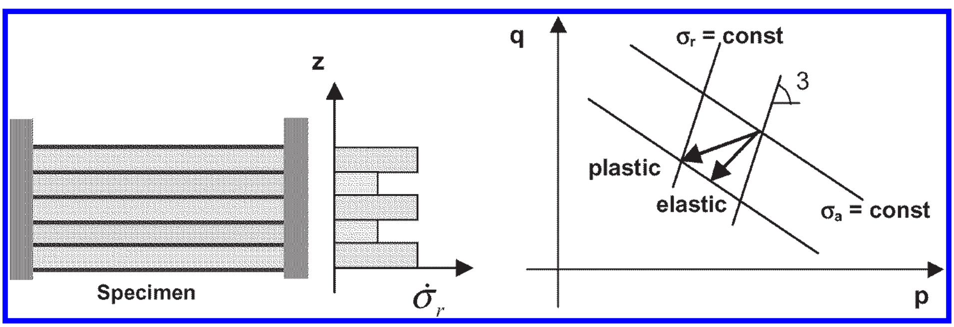

Figure 9. Bifurcation in an oedometric test on a cemented geomaterial with bond degradation a) compaction bands b) stress rates in the layers –After Nova et al. (2003)

The nullity of this stiffness coefficient, that coincides in this case with the occurrence of a peak in the axial stress-strain relationship (horizontal compaction band), is therefore the condition for bifurcation.

Softening can occur in unexpected conditions, therefore. As a final example, consider the results of an oedometric test on a specimen of loose sand, shown in Figure 10. During virgin loading, the ratio between horizontal and vertical stresses remains constant, so that the stress path in the plane p', q is a straight line passing through the origin. For moderate unloading, hysteretic unloading-reloading loops are apparent. If the material is fully unloaded, however, the reloading stress path is again a straight line, i.e. the material behaves as if it was virgin anew.

Such a result can be easily explained within the framework of the model. When unloading is moderate, initially the behavior is elastic while only limited plastic strains take place when the yield locus is reached again (for negative values of q). When unloading is large, on the contrary, after the initial elastic phase, large plastic strains take place and the material softens with a consequent reduction of the size of the elastic domain. If the specimen is fully unloaded, the elastic domain shrinks to a point, the material is in the same state as at the beginning of the test and the stress path upon reloading is that typical of a virgin specimen, i.e. a straight line passing through the origin. In a sense, we can say that the specimen 'fails' during unloading in an oedometric test, since softening is activated. This fact has no dramatic consequences, however, since the state of the material is simply driven back to a virgin condition.

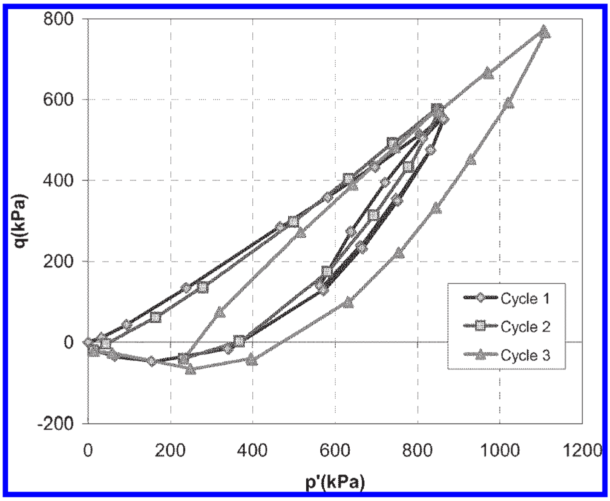

Figure 10. Experimental stress path in an oedometric test on a loose sand specimen, with cycles of unloading-reloading. After Castellanza (2002)

5 CONCLUSIONS

Failure of a material element is a primitive concept. When dealing with geomaterials such a concept becomes less neat, however. Although it is possible to define a convenient limit locus beyond which stress states are not feasible for a given material, in fact, unlimited strains may occur even for stress states within such locus, provided convenient loading conditions are imposed and controlling parameters appropriately chosen.

Such types of failures can occur in the hardening regime, provided the flow rule is non-associated and the stiffness matrix is not positive definite. An example is the loose sand specimen failure in a load controlled undrained test. Although under drained conditions the material is fully stable and shows a typical hardening behavior, if the kinematic and static control parameters are changed (e.g. from axial strain and confining pressure to volumetric strain and axial load), a brutal collapse takes place, for stress levels very far from the limit locus.

Other types of loss of control can occur. Whenever a principal minor of the stiffness (or compliance) matrix is zero, there exists a special combination of stress and strain increments for which an infinity of solutions, in terms of the complementary variables of strain and stress, is possible. Homogeneous bifurcations may take place, therefore.

It is also possible to consider linear combinations of stresses and strains. In this case a generalized stiffness matrix can be defined and a similar conclusion holds in terms of generalized stresses and strains. In particular, it can be shown that even the shear band type of bifurcation can be described